

P A R T I V

CULTURE AND  
LANGUAGE

## CHAPTER 21

# COGNITIVE LINGUISTICS AND THE CONCEPT(S) OF NUMBER

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## WHAT'S IN A NAME? THAT WHICH WE CALL A NUMBER

THE study of numerical cognition, as the name suggests, investigates the cognitive, psychological, developmental, and neural bases of *numbers*. The term ‘number,’ however, is highly polysemous. The meaning of ‘number’ in expressions such as ‘seven is a prime *number*’, ‘I forgot my passport *number*’, ‘not all languages mark *number* grammatically’, or ‘an infinitesimal *number* has non-Archimedean properties’, are related but resist capture by a single definition – in fact, some seem to differ radically from others. To make things more confusing, even in scholarly articles the term ‘number’ is sometimes used to mean ‘numeral’ (i.e. a sign for a specific number) as in ‘the big *number* 2 on the shirt with the giraffe’ (McMullen, 2010), and, more importantly, often it is used in lieu of ‘ numerosity’ – the numerical magnitude of stimuli arrays or designated collections – as in the title ‘spontaneous number representation in mosquitofish’ (Dadda, Piffer, Agrillo, & Bisazza, 2009). In the latter case, there is a risk of unwittingly ascribing ‘numerical’ properties, such as order or operativity to a far simpler ability to discriminate between stimuli. The mosquitofish is no mathematician. Yet such an ascription leads to the teleological argument that thousands of species, from fish to humans, have full-fledged ‘number’ representations as a result of biological evolution. Confusion must be avoided. So, what, then, is meant by *number* in numerical cognition?

In mathematics, numbers are abstract entities in their own right, governed by precise properties (e.g. for the real numbers, completeness). They are represented by specific signs – numerals, spoken or written words – and in prototypical cases, can be used to perform calculations. Depending on their properties, specific collections of these numbers are designated as *natural numbers*, *negative numbers*, *whole numbers*, *rational numbers*, *irrational numbers*, *real numbers*, *complex numbers*, *infinitesimal numbers*, *hyper-real numbers*, *surreal numbers*, *transfinite cardinal numbers*, and so on, all of which designate sets of ‘numbers.’ Some of these

are strict subsets of each other; others don't intersect at all. Within this vast universe of mathematical numbers, the field of 'numerical cognition' has, however, been concerned almost exclusively with *prototypical counting numbers* (PCNs) – 'small' and 'common' natural numbers, often less than 10 – not, as one might think, with *natural numbers* proper. Indeed, no study of numerical cognition has investigated natural numbers such as 745,987,345,316,472,995,084,821,410,488,458,390 – even though in mathematics, this number is as 'natural' as the number 4. The study of numerical cognition has, thus, taken 'number' to *primarily* mean PCNs, and from there has often, and inappropriately, we believe, advanced claims about the totality of natural numbers, integers, arithmetic, and even mathematical cognition in general. For instance, focusing on small counting numbers has led developmental psychologists to attribute knowledge of the natural numbers when children's knowledge was actually piecemeal, restricted to PCNs and even further to particular small numbers (Davidson, Eng, & Barner, 2012). It just so happens that PCNs are the most frequently manifested, observed, and (cognitively) investigated numbers, and happen to be the smallest natural numbers. However, since 'number' is polysemous in both everyday language and mathematics, this raises the question of how the various mathematical meanings of 'number' relate to each other and, in particular, of what cognitive mechanisms make possible the passage from PCNs to actual arithmetic – and eventually to mathematics. In this chapter, we will address these questions from the perspective of cognitive linguistics, a subdiscipline of linguistics, and cognitive science.

Cognitive linguistics developed in the 1980s with the work of Charles Fillmore (1982), Ron Langacker (1987), Len Talmy (1988), George Lakoff (Lakoff & Johnson, 1980), Gilles Fauconnier (1985), and others. Based on analyses of everyday language, cognitive linguistics has argued that language, rather than emerging from an innate domain-specific language faculty (e.g. Universal Grammar; Chomsky, 2007), is the product of domain-general cognitive mechanisms. Two well-studied cognitive linguistic phenomena are *conceptual metaphor* (Lakoff & Johnson, 1980) and *fictive motion* (Talmy, 1996). Conceptual metaphor is manifested in linguistic expressions where one domain is discussed in terms of another, as in the English expressions, 'Send her my warm hellos,' or 'He is a cold person' – expressions which discuss affection as having thermic properties, 'warm' when there is affection, 'cold' when there is a lack of it. Lakoff and Johnson (1980) argued that these systematic metaphorical expressions were the linguistic manifestation of underlying conceptualization – and thus, that metaphor was a phenomenon of *thought* and not just of *language*. Fictive motion is manifested in expressions such as, 'The fence *runs* along the river,' or 'The Equator *passes through* many countries,' in which real or imaginary static entities are discussed as if they were dynamic. Talmy (1996) argued that fictive motion in *language* may actually reflect a process of dynamic *conceptualization*. Over the last two decades, the cognitive reality of both conceptual metaphor and fictive motion has been supported by psychological experiments, gesture studies, EEG/ERP studies, and neuroimaging (for conceptual metaphor see Gibbs, 2008, for an overview; for fictive motion see Marghetis & Núñez, 2013; Matlock, 2010).

Inspired by the possibility that these two phenomena might be central to human imagination, Lakoff and Núñez (2000) argued that they play a role in the creation and conceptualization of number concepts, arithmetic, and mathematics in general. In this chapter, we review the proposal that cognitive linguistic mechanisms structure numerical cognition – including PCNs, but also the positive integers and arithmetic – and survey the empirical evidence that supports this proposal. The focus throughout is on reasoning, understanding,

and conceptualization – the way we think about numbers themselves, as abstract objects – rather than the psychological processing of particular numbers.

## FROM COUNTING TO ARITHMETIC (AND STILL FAR FROM MATHEMATICS)

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Counting is an apparently simple practice that many children around the world learn at an early age, usually based on a lexicon of ‘counting’ numbers, which, varying across the languages of the world (Beller & Bender, 2008), refer to prototypical quantities (‘one,’ ‘two,’ ‘three,’ ‘four,’ etc.). Two questions then immediately come up. First, is this apparently simple act a human universal? If not, what does that tell us about the nature of numbers as a whole, and about the nature of the simplest PCNs, in particular? Secondly, how do we get from ‘counting’ to PCNs, to natural numbers, to arithmetic, and to mathematics proper?

It is well established that many languages around the world do not have words for exact numerosities beyond ‘four’ or ‘five’ (Strömer, 1932), and that native speakers of these languages seem to reason only approximately about numerosities above that range (Pica, Lemer, Izard, & Dehaene, 2004). The apparently simple act of counting beyond, say half a dozen items, is not part of our biological endowment. That is, numbers of all sorts – PCNs, but also natural, negative, irrational, real, complex, infinitesimal, transfinite – have not been selected for by natural selection, but rather emerged out of specific cultural practices that developed over historical time. To get beyond our basic cognitive machinery, therefore, we need an account of how abstract concepts can acquire complex structure – a process that, we suggest, is realized via mechanisms like conceptual metaphor and fictive motion. However, there is more. All languages around the world seem to have a precise number lexicon for at least the most basic numerosities ‘one,’ ‘two,’ and ‘three’ (or at least ‘one’ and ‘two;’ Gordon, 2004). What this suggests is that count practices and PCNs seem to build on basic *cognitive pre-conditions for numerical abilities* (Núñez, 2009) that, although not being *numerical* as such, are part of the human biological endowment and are, therefore, universal. One such pre-condition is *subitizing*, the capacity to make quick, error-free, and precise judgments of the numerosity of small collections of items (Kaufmann, Lord, Reese, & Volkmann, 1949), which seems to be based on domain general mechanisms that deal with multiple object individuation (Piazza, Fumarola, Chinello, & Melcher, 2011). Humans can subitize up to about three to four items. Findings suggest that the ability to subitize is innate, that it exists even in some amphibians (Uller, Jaeger, Guidry, & Martin, 2003; see also chapter by Agrillo, this volume), that it is not merely a pattern-recognition process, and that it is not restricted to the visual modality but also operates on, e.g. sequences of knocks or beeps (Davis & Pérusse, 1988). Moreover, deficits in subitizing seem to be associated with developmental dyscalculia (Moeller, Neuburger, Kaufmann, Landerl, & Nuerk, 2009), suggesting an influence on the consolidation of numerical capabilities. Another innate pre-condition is the ability to discriminate, approximately, between large collections on the basis of numerosity – an ability that emerges in humans as early as 6 months, provided the collections’ numerosities differ by a sufficiently large ratio (e.g. 8 versus 16, but not 8 versus 12; Xu & Spelke, 2000). Although these inborn abilities exist independently of culture, education, and linguistic practices

(Wiese, 2003), the findings are consistent with the fact that all human languages have lexical items that capture the meaningful experience of discriminating small, precise numerosities (provided by subitizing), and others that express the approximate relations (greater/smaller than) involved in large numerosity discrimination. Language, in a uniquely human form, consolidates at a higher symbolic level these inborn cognitive pre-conditions for numerical abilities, which we share with many other species of the animal kingdom. Subitizing and large numerosity discrimination, however, are not numerical proper, let alone arithmetical or mathematical. They are simply capacities for discriminating numerosities, which lack essential components of *number* concepts. The essential notion of *order*, for instance, does not appear in humans until as late as 11 months (Brannon, 2002). Most importantly, these basic capacities also lack compositionality, the core of the creation of proper numbers, arithmetic, and mathematics. Thus, these innate cognitive preconditions for PCNs are:

- (1) insufficient for number at the level of complexity of natural numbers, and
- (2) not ‘proto’ or ‘early’ *numerical* any more than the vestibular system supporting an infant’s first steps is proto – or early snowboarding (Núñez, 2009).

Innate cognitive preconditions cannot provide foundations for the precision, richness, and range of PCNs, let alone for more sophisticated number concepts, arithmetic, and complex mathematical concepts.

Any account of *number cognition* must account for the unique features of PCNs, natural numbers and arithmetic – things like precision, objectivity, rigor, generalizability, stability, and symbolism. These concepts are highly sophisticated and developed culturally only in recent human history. How to get from our basic cognitive toolbox to these rich cultural accomplishments? One possibility is that PCNs, natural numbers and arithmetic are realized through precise combinations of domain-general (i.e. non-mathematical) everyday cognitive mechanisms that make human imagination and abstraction possible, such as conceptual metaphor and fictive motion (Lakoff & Núñez, 2000; Núñez, 2009). According to this proposal, these cognitive mechanisms – mediated through language and other cultural practices – support the conceptualization of precise abstract entities like number and arithmetic, grounding them in our embodied experience<sup>2</sup>.

## CONCEPTUALIZING NUMBER: CONCEPTUAL METAPHOR AND FICTIVE MOTION

In this section we analyse how conceptual mappings – conceptual metaphor and fictive motion in particular – provide the inferential structure of number concepts. We begin with how these mechanisms work in everyday language.

### Introduction to Conceptual Mappings and Fictive Motion

#### *Conceptual Metaphor*

Consider the following two everyday linguistic expressions: ‘The election is *ahead* of us,’ and, ‘Winter is now *behind* us.’ Even though ‘an election’ is not physically ‘ahead’ of us, and

'Winter' is not literally 'behind' us, these expressions are, nevertheless, easily understood to convey a precise meaning – namely, a *temporal* meaning. Countless such expressions, whose meaning is not literal, but *metaphorical*, can be observed in human everyday language in many domains. 'Metaphor,' in this sense, is not just a figure of speech, or a rhetorical tool reserved for poets and politicians. It is a mechanism of thought, usually operating unconsciously and effortlessly, but ubiquitous in everyday (and technical) language (Lakoff & Johnson, 1980).

Cognitive linguistics has shown that these metaphorical expressions are systematic, such that the countless metaphorical expressions can be modelled by a relatively small number of conceptual metaphors. By 'conceptual metaphor' we mean both a particular inference-preserving cross-domain mapping, and also the cognitive mechanism that enables such mappings. Conceptual metaphor involves projecting the inferential structure of a grounded *source domain* (e.g. spatial experience) to a different *target domain*, usually more abstract (e.g. time). In the above examples, specific notions related to sagittal bodily space like 'ahead' and 'behind' get mapped onto 'future' and 'past', respectively, and open up an entire world of inferences where the relatively abstract domain of 'time' is conceived in terms of the more concrete domain of spatial experience. Crucially, conceptual metaphor can account for abstract domains' inferential organization: the network of inferences<sup>3</sup> that is generated via the mappings (see Table 21.1).

Notice that, although the expressions in our initial example use completely different *words* (i.e. 'ahead,' 'behind'), they are both linguistic manifestations of a single general conceptual metaphor, namely, TIME EVENTS ARE LOCATIONS IN SAGITTAL UNI-DIMENSIONAL SPACE<sup>4</sup>. As in any conceptual metaphor, the inferential structure of target domain concepts (time, in this case) is created via a precise mapping from the source domain (in this case sagittal uni-dimensional space – the linear space in front and behind an observer). The inferential structure of this mapping accounts for a number of linguistic expressions, such as 'The summer is still *far away*', 'The end of the world is *near*', and 'Election day is *here*'. Many important entailments follow from the mapping. For instance, transitive properties applying to spatial relations between the observer and the objects in the source domain are preserved in the target domain of time: if, relative to the front of the observer, object A is further away than object B, and object B is further away than object C, then object C is closer than object A. Via the mapping, this implies that time C is in a 'nearer' future than time A. The same

**Table 21.1 The time events are locations in sagittal uni-dimensional space metaphor**

Source Domain	Target domain
Sagittal uni-dimensional space relative to ego	Time
Locations in front of ego	→ Future times
Locations behind ego	→ Past times
Co-location with ego	→ Present time
Farther away in front of ego	→ 'Farther away' in the future
Farther away behind ego	→ 'Farther away' in the past

relationships hold for objects behind the observer and times in the past. Furthermore, the conceptual mapping can inject novel structure into the target domain. Time, for instance, is seen as having measurable ‘extension’ and as extending like a segment of a path, and is thus conceivable as a linear bounded region.

### *Fictive Motion*

Fictive motion is a cognitive mechanism through which we unconsciously (and effortlessly) conceptualize static entities in dynamic terms, as when we say, ‘The road goes along the coast.’ The road itself doesn’t actually move anywhere. It is simply standing still. But we may conceive it as moving ‘along the coast.’ Len Talmy (1996) analysed linguistic expressions taken from everyday language in which static scenes are described in dynamic terms. Consider:

- The fence *stops* right after the tree.
- The Equator *passes through* many countries.
- The border between Switzerland and Germany *runs along* the Rhine.
- The California coast *goes all the way down* to San Diego

Motion, in these cases, is fictive, imaginary, not real in any literal sense. Not only do these expressions recruit verbs of action, but they provide precise descriptions of the quality, manner, and form of motion. In all cases of fictive motion there is a *trajector* (the moving agent) and a *landscape* (the construed space in which the trajector moves). The trajector can be a physical object (e.g. ‘the road goes;’ ‘the fence stops’), or a social, imagined, or metaphorical object (e.g. ‘the Equator passes through;’ ‘the border runs’). In many cases of fictive motion, real world trajectories don’t move, but are associated with something that could; we can talk about a ‘road running from Berkeley to San Diego,’ and roads are associated with cars, which *can* move. As we will see, this is never the case in mathematics, where the motion of the trajector is always metaphorical: we talk of *oscillating* functions or *approaching* a limit, in the absence of any concrete objects that could move (Marghetis & Núñez, 2013; Núñez, 2006).

## CONCEPTUAL MAPPINGS AND FICTIVE MOTION IN THE CONCEPTUALIZATION OF NUMBERS AND ARITHMETIC

As discussed in the previous section, numbers systems and arithmetic are qualitatively more complex than subitizing or judgments of approximate numerosity, the preconditions for numerical abilities that are found in monkeys and human infants. Subitizing is exact within its range, but natural numbers are always exact; approximate judgments of numerosity apply to collections of all sizes, but are inherently *approximate*. From where, then, do PCNs (let alone natural numbers) get their properties? What cognitive mechanisms are needed to go from simple innate abilities to full-blown arithmetic?

Humans seem capable of transcending the limitations of these simple abilities. When sociocultural demands are present, counting emerges. In order to count (e.g. finger-counting), several additional capacities are required – capacities like grouping objects into real or imagined collections, ordering these collections for serially counting, assigning number-labels to individual objects, and using memory to keep track of this process, amongst others (Lakoff & Núñez, 2000)<sup>5</sup>. When these capacities are used within the subitizing range (between 1 and 4), stable results are obtained because cardinal-number assignment is co-extensive with outcomes obtained via subitizing. To count beyond four – the range of the subitizing capacity – requires the above cognitive capacities, but also additional capacities that allow for putting together perceived or imagined groups to form larger groups, and a capacity to associate physical symbols (or words) with the resulting exact quantities. However, subitizing and counting only provide some of the cognitive preconditions for mature numerical abilities, and additional mechanisms are required to go beyond this foundation – imaginative mechanisms like conceptual metaphor and fictive motion. These mechanisms allow for the conceptualization of cardinal numbers and arithmetic operations in terms of basic ordinary experiences of various kinds – experiences with groups of objects, with the part–whole structure of objects, with distances, with movement and locations, and so on. Conceptual metaphor and fictive motion are among the most basic domain-general everyday cognitive mechanisms that take us beyond minimal early abilities and simple counting to the elementary arithmetic of PCNs and natural numbers.

Since conceptual metaphors preserve inferential organization, they make possible the conceptualization of arithmetic in terms of the prior understanding of commonplace physical activities. The understanding of elementary arithmetic seems to be based on a systematic correlation between (1) the most basic literal aspects of arithmetic, such as subitizing and counting, and (2) everyday activities, such as collecting objects into groups or piles, taking objects apart and putting them together, taking steps on a path, and so on. Such correlations allow humans – unlike other animals – to form conceptual mappings by which they greatly extend their subitizing and counting capacities. Thus, if we conceptualize numbers as collections or as locations in space, we can project the logic of collections and of spatial locations, respectively, onto numbers thus providing their inferential structure.

On the basis of these considerations, Lakoff and Núñez (2000) suggested that two fundamental conceptual metaphors are responsible for number concepts and arithmetic: **ARITHMETIC IS OBJECT COLLECTION** and **ARITHMETIC IS MOTION ALONG A PATH**. These metaphors are a way to ground our conceptualization of arithmetic in shared, precise bodily experiences, and thus provide the necessary inferential organization. The detailed analysis of these mappings can be found elsewhere (Lakoff and Núñez, 2000, chapters 3 and 4). In order to give a flavour of the robustness and inferential richness of these conceptual mappings, let us point to some of their crucial components.

The **ARITHMETIC IS OBJECT COLLECTION** metaphor is a mapping from the domain of physical objects to the domain of numbers. The metaphorical mapping consists of:

- (1) The source domain of object collection (based on our experiences with grouping objects).
- (2) The target domain of arithmetic of PCNs (structured non-metaphorically by subitizing and counting).

**Table 21.2 The arithmetic is object collection metaphor**

Source Domain	Target domain
<b>Object collection</b>	<b>Arithmetic of PCNs</b>
Collections of objects of the same size	→ Numbers
The size of the collection	→ The 'size' of the number
The smallest collection	→ The unit (one)
Bigger	→ Greater than
Smaller	→ Less than
Putting collections together	→ Addition
Taking a smaller collection from a larger collection	→ Subtraction

- (3) A mapping across the domains (based on our experience subitizing and counting objects in groups). The basic mapping of this conceptual metaphor is the following (Table 21.2):

Evidence of this metaphor shows up in everyday language. The word ‘add’ has the physical meaning of placing a substance or a number of objects into a container (or group of objects), as in ‘add sugar to my coffee,’ ‘add some logs to the fire,’ and ‘add onions and carrots to the soup.’ Similarly, we can ‘take 7 from 10,’ ‘take 3 out of 4,’ or ‘take away 2’ – but ‘take ... from,’ ‘take ... out of,’ and ‘take away ...’ have the physical meaning of removing a substance, an object, or a number of objects from some container or collection. Linguistic examples include ‘take some water from this pot,’ ‘take some books out of the box,’ and ‘take away some of these logs.’ On our proposal, these linguistic regularities are neither random nor superficial. They reflect the recycling of inferences about real-world actions, such as ‘to add’ and ‘to take away,’ recruited via the metaphor to conceptualize abstract arithmetic facts. It follows from the metaphor that adding yields something bigger (more) and subtracting yields something smaller (less). Accordingly, lexical items like ‘big’ and ‘small,’ which indicate literal size for objects and collections of objects, are metaphorically extended so they apply to numbers, as in ‘which is bigger, 5 or 7?’ and ‘two is smaller than four.’ However, this isn’t the only way to conceptualize arithmetic. Complementing the OBJECT COLLECTION metaphor is the ARITHMETIC IS MOTION ALONG A PATH metaphor. The metaphorical mapping consists of:

- (1) The source domain of paths (based on shared experiences with motion through linear space).
- (2) The target domain of arithmetic of PCNs plus Zero (structured non-metaphorically by subitizing and counting).
- (3) A mapping across the domains (based on our experience subitizing and counting steps on a path). The basic mapping is the following (Table 21.3):

**Table 21.3 The arithmetic is motion along a path metaphor**

Source domain	Target domain
<b>Motion along a path</b>	<b>Arithmetic of PCNs (plus 0)</b>
Point-locations on a path	→ Numbers
The origin, the beginning of the path	→ Zero
A point-location on the path	→ One
Farther from the origin than	→ Greater than
Closer to the origin than	→ Less than
The smallest collection	→ The unit (one)
Acts of moving along the path	→ Arithmetic operations
Moving from a point-location <i>A</i> away from the origin, a distance that is the same as the distance from the origin to a point-location <i>B</i>	→ Addition of <i>B</i> to <i>A</i>
Moving toward the origin from <i>A</i> , a distance that is the same as the distance from the origin to <i>B</i>	→ Subtraction of <i>B</i> from <i>A</i>

This understanding of numbers as point-locations is expressed linguistically in a variety of ways:

- ‘How *close* are these two numbers?’
- ‘37 is *far away* from 189,712.’
- ‘8.9 is *near* 9.’
- ‘The result is *around* 50.’
- ‘Count *up* to 20, without *skipping* any numbers.’
- ‘Count *backward* from 20.’
- ‘Count *to* 100, *starting at* 20.’
- ‘Name all the numbers *from* 2 to 10.’

Note that this metaphor relies additionally on fictive motion, with arithmetic operations conceptualized dynamically as individual acts of motion along a path.

These conceptual metaphors are rich in their inferential entailments, derived from basic facts about collections of physical objects or locations on paths. The result is a set of inferences about PCNs, only later extended to the natural numbers (Davidson et al., 2012). For example, suppose we have two collections, *A* and *B*, of physical objects, with *A* bigger than *B*. What happens if we add the same collection *C* to each? Well, *A* plus *C* will be a bigger collection of physical objects than *B* plus *C*. This is a fact about collections of physical objects of the same size. Using the mapping ARITHMETIC IS OBJECT COLLECTION, this physical fact that we experience in grouping objects can now be conceptualized as an arithmetical truth about *numbers*: if a number *A* is greater than number *B*, then *A* plus number *C* is greater than *B* plus *C*. Similar metaphorical analyses are available for various ‘truths’ of arithmetic, including such things as stability of results for addition and subtraction, closure

for addition, the existence of inverse operations, the possibility of unlimited iteration for addition but not for subtraction, and so on (Lakoff & Núñez, 2000, chapters 3 and 4). In each case, the conceptual metaphor maps a property from the source domain of object collections or motion along a path, to the target domain of numbers and arithmetic. On this account, the laws and truths of arithmetic are, in fact, metaphorical entailments of the conceptual mapping we operate with. The truths seem certain because the metaphors are invisible, but formative of our number cognition; we all agree because these metaphors are shared.

The inferential organization of these metaphorical mappings supplies a wide range of essential arithmetic properties. For instance, several *equational properties* that apply to object collections and motion along a path, respectively, get metaphorically extended to numbers:

- *Equality of result*: different operations produce the same resulting object collection or landing location on a path.
- *Preservation of equality*: combining equals to equals yields equals; likewise for subtraction
- *Commutativity*: combining collections is order-independent, as are forward motions along a path.

Moreover, several *relationship properties* are also preserved by the metaphorical mappings:

- *Duality of ordering*: if  $A$  is greater than  $B$ , then  $B$  is less than  $A$ .
- *Trichotomous ordering*: if  $A$  and  $B$  are two collection/numbers or locations on a path, then either  $A$  is greater than  $B$ , or  $B$  is greater than  $A$ , or  $A$  and  $B$  are of the same magnitude.
- *Symmetry of equality*: if  $A$  is the same size as  $B$ , then  $B$  is the same size as  $A$ .
- *Transitivity*: if  $A$  is greater than  $B$  and  $B$  is greater than  $C$ , then  $A$  is greater than  $C$ .

These powerful conceptual metaphors have many more entailments, and lend themselves to further extensions. Within the domain of object collection, for instance, we can engage in pooling and repeated addition of object collections, as well as splitting and repeated subtraction. It is possible then to state precisely how, via the metaphorical mappings, these entailments in the realm of object collection and motions along a path give rise to arithmetical laws for multiplication and division of natural numbers, including commutativity and associativity for multiplication, distributivity for multiplication over addition, and the existence of multiplicative identity and inverse. The details of this analysis, however, exceed the space available here (for details, see Lakoff & Núñez 2000, chapter 3).

In this brief analysis, one can see the extent to which the two conceptual metaphors ARITHMETIC IS OBJECT COLLECTION and ARITHMETIC IS MOTION ALONG A PATH ground the rich inferential structure of PCN concepts. A crucial component of the robustness of the grounded inferential structure comes from the close agreement in the inferential structure of the two source domains – collections and motion along a path. Many actions in one source domain (e.g. collecting objects) have a corresponding action in the other source domain (e.g. moving along a path), with both actions mapping to identical results in the target domain of numbers. In fact, in the realm of PCNs (without the concept ZERO) and basic

arithmetic operations, there is an *isomorphism* between the source domains, thus securing solid foundations of numbers and arithmetic in everyday embodied experience.

These two conceptual metaphors, however, differ in inferential and cognitive complexity. Inferentially, the ARITHMETIC IS OBJECT COLLECTION metaphor does not have a number zero, for instance. Strictly speaking, the result of taking away collection *A* from collection *A* yields a *lack* of collection. It takes extra metaphorical extensions to make the lack of a collection to ‘be’ a ‘collection with no elements.’ ARITHMETIC IS MOTION ALONG A PATH, on the other hand, comes with a natural ‘built-in’ number zero (the origin of motion), which, via the mapping, gets the same ontological status as the rest of PCNs. Zero is a location on the path, and thus a number like any other. Moreover, this metaphor readily affords a further extension to negative numbers, which the object collection metaphor cannot provide. And as we’ll argue in the next section, there is now strong evidence suggesting that ARITHMETIC IS OBJECT COLLECTION is more easily acquired and culturally universal than ARITHMETIC IS MOTION ALONG A PATH (Núñez, 2009; Núñez, Cooperrider, & Wassman, 2012a).

A metaphorical analysis suggests that many of the basic properties of arithmetic can be derived from shared, embodied experiences – collecting discrete objects and moving along a path. However, this theoretical account – based primarily on evidence from linguistic practices – says nothing about when and to what end these metaphors are used during online reasoning. For that, we must leave the armchair and seek empirical evidence.

## EMPIRICAL SUPPORT FOR THE EMBODIED CONCEPTUALIZATION OF NUMBER

The foregoing analysis grew out of a linguistic tradition that relied primarily on regularities in figurative language. This linguistic evidence, however, cannot tell us when or in what contexts embodied reasoning makes its contribution, if at all. Linguistic evidence on its own does not provide a transparent window into conceptual structure (Murphy, 1996) and is, therefore, insufficient to demonstrate the cognitive reality of metaphorical thought; that we speak of death as ‘kicking the bucket’ is weak evidence that we conceptualize death in terms of boots and buckets (Casasanto, 2009). We can reason about arithmetic, moreover, solely by manipulating abstract notations, thus taking advantage of external space to structure the intelligent manipulation of inscriptions (Kirsh, 1995; de Cruz, 2008; Landy & Goldstone, 2009; cf. Rumelhart, Smolensky, McClelland, & Hinton, 1986). So, metaphorical thought cannot be the whole story of how we reason about numbers and arithmetic. It’s an empirical project, therefore, to determine exactly when, why, and for what purposes we engage in metaphorical thought about number and arithmetic.

In the last decade, research on embodied cognition has begun that project, using an increasingly diverse toolkit of empirical methods to document the functional role of embodied thought in language and cognition more generally (Gibbs, 2006). There is now converging evidence that number cognition is, indeed, embodied and metaphorical, at least sometimes and under certain circumstances. This section surveys that evidence.

## What Roles for Imagination and Conceptualization?

An embodied approach to PCNs, and to number cognition in general, needs to distinguish between the many possible contributions that metaphorical thought could make to arithmetic reasoning (cf. Gibbs, 1994, p. 18). Consider the following hypotheses:

- H1: Embodied metaphors show up in language, but have no cognitive reality.
- H2: Embodied metaphors play a role in learning.
- H3: Embodied metaphors are activated during online reasoning.

The first hypothesis was the prevailing view for most of history, dating back to Aristotle, and is a reasonable working hypothesis (Lakoff, 1993). Indeed, there are certainly linguistic metaphors that no longer reflect metaphorical conceptualization, linguistic metaphors that have become *dead* metaphors. The term ‘*until*’, for instance, has lost the spatial meaning it had in Middle English, inherited from Old Norse (Traugott, 1985). This could be true of mathematics, so that expressions like ‘*bigger* and *smaller* numbers’ and ‘keep adding numbers until you *pass* one hundred’ are merely the sedimentation of mathematical history and uninformative about contemporary number cognition.

The last few decades of research in cognitive linguistics, however, has demonstrated that where there’s linguistic smoke, there’s often cognitive fire. According to the second hypothesis, metaphorical thought may show up during concept acquisition and in the classroom, but not necessarily outside of that pedagogical context. After all, physicist and Nobel Laureate Richard Feynman claimed that, ‘Light is something like raindrops’ (1985, p. 14), but presumably did not conceptualize electromagnetic radiation as small wet blobs.

The third hypothesis goes further to suggest that embodied conceptual resources may play a functional role in mature reasoning about number and arithmetic. Consider the task of understanding why the sum of two even integers is itself even. There exist technical proofs of this fact – proofs that rely on algebraic notation – but conceptualizing arithmetic as either *Object Collection* or as *Motion Along A Path* might afford a more grounded, experiential insight into the behaviour of PCNs, and other numbers in general. In cases like these, we might expect embodied thought to guide reasoning.

These latter two hypotheses are not meant to be mutually exclusive, but focus attention on two compelling proposals for how conceptual metaphor and fictive motion might actually influence number cognition. In what follows, we address these two hypotheses in turn.

## Embodied Learning

The first possibility is that metaphorical reasoning appears during learning, supporting the early acquisition and later elaboration of number concepts. Right from the very beginning, acquiring the concept of EXACT NUMBER seems to depend on manipulating collections of objects. Developmental psychologists disagree about the details, but most agree that the acquisition process relies on the child’s ability to apply a counting routine to collections of discrete objects (Carey, 2009; Spelke & Tsvirkina, 2001; but see Rips, Bloomfield, & Asmuth, 2008). According to Carey (2009), for instance, learning the meaning of the number lexicon (‘one,’ ‘two,’ etc.) relies on early experiences with small collections of objects, extended via analogy to all the positive counting numbers as a result of applying a count routine to varied

sets of objects (Carey, 2009). The child's first concept of number, therefore, is intimately tied to experiences of manipulating collections, the source domain of the OBJECT COLLECTION metaphor. The formative influence of this source domain is reflected in children's early difficulties with arithmetic. Hughes (1986) reports that 3- and 4-year old children can accurately add and subtract small numbers when the numbers are used to describe collections of perceived or imagined objects, such as bricks being placed in a box. However, when the number words are used in isolation, without a concrete context, children reliably fail to respond correctly. The ARITHMETIC IS OBJECT COLLECTION metaphor, therefore, seems central to the acquisition and early use of the concept of exact number.

Conceptualizing arithmetic as MOTION ALONG A PATH, on the other hand, does not seem to emerge until later in development, and requires extensive cultural scaffolding. Earlier research suggested that mapping numbers to space might be a cultural universal. Dehaene, Izard, Spelke, & Pica (2008) reported that an Amazonian indigenous group, the Mundurucu, were able to reliably and systematically map numbers to locations along a physical line segment, even though their language has a limited number lexicon, they lack formal education, and make little or no use of cultural artifacts like rulers, graphs, or maps. On the basis of this surprising result, they concluded that 'the mapping of numbers onto space is a universal intuition' (p. 1217). Recent research suggests that this conclusion may have been premature. Núñez, et al. (2012a) found that the Yupno of Papua New Guinea – who, unlike the Mundurucu, have an elaborate number lexicon, including words for exact numbers beyond twenty – did not map numbers to locations along a physical line segment. Instead, they systematically mapped small numbers to one endpoint and mid-size and large numbers to the other, ignoring the extent of the line segment. This categorical response pattern persisted even when the instructions explicitly described the line segment as a path, and explicitly demonstrated that a mid-sized numerosity was associated with a location around the mid-point of the path. These Yupno findings suggest that mapping between number and continuous space is not innate, does not emerge spontaneously, varies across cultures, and may require extensive cultural scaffolding (Núñez, 2011).

That scaffolding could be provided by rulers, calendars, graphs, and countless other cultural artifacts that map numbers to locations in space. It is a common pedagogical practice to introduce a material representation of a path-based construal of arithmetic: a literal 'number-line' with locations along a line segment labeled with the integers (Herbst, 1997). The ubiquity of such representations creates a cultural milieu in which associations between number and space are unavoidable, thus supporting the conceptualization of numbers and arithmetic as MOTION ALONG A PATH (Núñez, 2011). As expected, children raised in such a milieu gradually learn to map numbers to locations along a line. If North American children are shown a physical line segment with '0' at one endpoint and a numeral representing a larger number on the other (e.g. '100'), as early as kindergarten they are able to place intermediate numbers along the line segment in a systematic way (Siegler & Booth, 2004) – much like the reported Mundurucu, but unlike the Yupno. North American children's facility with this number-to-line mapping increases across development, and children gradually transition from a logarithmic mapping – allocating more space to smaller numbers – to a linear mapping, so that by the sixth grade children use a linear mapping to map numbers between 0 and 1000 to a line segment (Siegler & Opfer, 2003). Importantly, however, the logarithmic mapping persists even up to college age if participants report quantities with unconventional non-spatial methods (e.g. by squeezing or vocalizing), suggesting that

linear mappings are the result of training and education with culturally-developed notational devices like the physical number-line (Núñez, Doan, & Nikoulina, 2011). However, does acquiring these metaphors affect long-term learning outcomes? It seems plausible that it could. Conceptual metaphors ground the understanding of arithmetic in more concrete, experiential domains, and thus are a way to ground mathematical reasoning in shared, embodied intuitions – intuitions about motion through space, for instance, or about manipulating objects (Núñez, Edwards, & Matos, 1999). Inspired by this possibility, Danesi (2003, 2007) used *Conceptual Metaphor Theory* as a pedagogical tool for teaching mathematical reasoning to eighth-grade students with difficulties in mathematics. In one exploratory study, eighth-grade students were explicitly taught to identify conceptual metaphors in algebra word problems. At the beginning of the school year, the students had ‘severe difficulties in solving word problems’; by the end of the year, these at-risk students had closed the gap with their peers. While these preliminary studies require follow-up with fully controlled studies, nevertheless, they reinforce the promise of metaphor-based interventions in education – an area ripe for investigation.

Furthermore, long-term mathematical success depends on the acquisition of a linear spatial construal of number. In a longitudinal study of mathematics learning, Gunderson, Ramirez, Beilock, and Levine (2012) followed first – and second – grade students and, for five 4-month intervals, measuring their spatial abilities, symbolic mathematical abilities, and ability to associate numbers with locations along a line segment using a linear mapping. Students’ early spatial abilities were predictive of their end-of-year abilities in symbolic arithmetic, replicating previous findings (e.g. Kurdek & Sinclair, 2001; Lachance & Mazzocco, 2006). Crucially, however, this was mediated by their ability to map numbers to a line using a linear mapping. Learning to think of numbers as evenly distributed locations along a path (i.e. preserving magnitude and, therefore, spatial invariance of the increment between the successor and the predecessor of any given PCN) facilitates the acquisition of symbolic mathematical proficiency.

Both PCN concept acquisition and the transition to more advanced mathematics, including algebra word problems, seems to rely on learning to map between PCNs and more concrete domains, either collections of objects or locations along a linear path. These two conceptual metaphors, therefore, seem to play a role in mathematics learning, as suggested by Hypothesis 2. Once acquired, though, do these conceptual metaphors remain active, or are they discarded in favour of more abstract modes of reasoning?

## Embodied Reasoning

When Lakoff and Núñez (2000) first suggested that numbers and arithmetic were conceptualized as OBJECT COLLECTION or as MOTION ALONG A PATH, there was little evidence that these metaphors were active during online number cognition. The last decade, however, has begun to produce converging evidence that mathematical metaphors are not mere pedagogical scaffolds, but retain their psychological reality and are activated during mathematical thought.

If the ARITHMETIC IS MOTION ALONG A PATH metaphor is active during mature number cognition, then thinking about numbers should systematically activate linear spatial schemas, and thinking about arithmetic, motion through space. This is precisely what has

been observed. Processing PCNs can bias the spatial trajectory of subsequent responses. In a visual search task, task-irrelevant numbers prime subsequent eye movements, with smaller numbers priming left eye movements and larger numbers priming right eye movements (Fischer, Castel, Dodd, & Pratt, 2003). These effects also go in the other direction, from space to number. When participants were asked to generate random numbers while shaking their head back and forth, head position systematically biased the magnitude of the ‘random’ number, with smaller numbers generated when facing leftward, larger numbers when facing rightward (Loetscher, Schwarz, Schubiger, & Brugger, 2008); an analogous effect exists for shifts in eye gaze (Loetscher, Bockisch, Nicholls, & Brugger, 2010). So, task-irrelevant numbers and spatial orientation can bias each other in both directions.

Perhaps the best-known behavioural phenomenon is the Spatial-Numerical Association of Response Codes, or SNARC effect. Dehaene, Bossini, & Giraux (1993) had participants judge the relative magnitude (‘Greater or less than 5?’) or parity (‘Even or odd?’) of numbers between 1 and 9, and respond by pressing one of two buttons. Participants were faster to respond to smaller numbers when they were responding on the left, and to larger numbers on the right, as if participants were automatically activating a mental ‘number-line.’ Crucially, the SNARC shows up for non-manual responses (Schwarz & Keus, 2004; Schwarz & Müller, 2006), so it is not effector-specific – it is not, for instance, merely the result of experience with keyboards that display numbers from left to right.

On the basis of this phenomenon, and other evidence of number-space associations (see Hubbard, Piazza, Pinel, & Dehaene, 2005; van Dijck et al., this volume), some authors have concluded that ‘the representation of [...] number magnitude [...] is spatial in nature’ (Treccani & Umiltà, 2010). Note that these authors are *not* claiming that number magnitude is *mapped* to space, guided by culture and mechanisms of creative cognition. Instead, their claim is that number magnitude just *is* spatial, at least as represented in the brain. There are reasons to doubt this stronger claim. For one, the SNARC effect is shaped by lifelong participation in cultural practices; the orientation of the SNARC effect – left-to-right or right-to-left – is influenced by reading direction for words (Shaki & Fischer, 2008) and numbers (Shaki, Fischer, & Petrusic, 2009), and perhaps finger-counting practices (Fischer, 2008; Lindemann, Alipour, & Fischer, 2011). Moreover, we recently found that number magnitude is just as easily associated with *pitch* as it is with space (Marghetis, Walker, Bergen, & Núñez, 2011). Following Dehaene et al (1993), we had participants make relative magnitude judgments for numbers between 1 and 9, but with one difference: instead of responding spatially by pressing a left or right button, participants responded vocally by producing high- or low-pitched vocalizations (‘Ahhh’). Much like the SNARC effect, we found an interaction between number magnitude and pitch – participants responded faster to ‘lower’ numbers if they had to respond with a *low pitch*, but faster to ‘higher’ numbers with a *high pitch*. It seems, then, that participants were *Seeing Number As Pitch* – a SNAP effect. Perhaps number magnitude is both spatial *and* pitch-based, but we find that unlikely. Rather, the SNAP effect seems to reflect the human capacity to rapidly and unconsciously map between conceptual domains<sup>6</sup> (cf. Fauconnier & Turner, 2002; Lakoff & Johnson, 1980). These considerations suggest that the SNARC and other effects are not evidence that ‘number magnitude is spatial,’ but reflect a learned mapping from numbers to space, shaped by culture, context, and embodied experience.

Much like number, arithmetic interacts with space in ways consistent with a metaphorical analysis. Recent studies suggest that arithmetic may involve shifts in attention along a spatial representation of number. In one study (McCrink, Dehaene, & Dehaene-Lambertz, 2007),

subjects saw two collections of dots that were added (i.e. combined) or subtracted behind an occluding screen. The screen then disappeared to reveal a third collection, and subjects had to judge whether it had the correct number of dots. Responses on this verification task were systematically biased by the arithmetic operation, such that subjects were more likely to accept a collection with too many dots after addition, but more likely to accept a collection with too few dots after subtraction. Although space was not directly implicated in their study, McCrink and colleagues interpreted this ‘Operational Momentum’ effect as evidence that participants were ‘overshooting’ as they shifted their attention along a mental number-line. Operational Momentum is not restricted to approximate arithmetic, but has also been observed with symbolic arithmetic, such as adding and subtracting single digit numbers (Pinhas & Fischer, 2008). Finally, it appears that arithmetic calculation shares a neural substrate with brain areas responsible for shifts in spatial attention, with leftward shifts in attention associated with subtraction and rightward shifts in attention associated with addition (Knops, Thirion, Hubbard, Michel, & Dehaene, 2009). Taken together, the SNARC and Operational Momentum effects demonstrate that online reasoning about number and arithmetic automatically activates spatial representations, exactly as expected if the ARITHMETIC IS MOTION ALONG A PATH metaphor plays an enduring role in number cognition.

More recently, we have turned to the study of gesture – the spontaneous bodily actions that we all co-produce while speaking or thinking, most often involving the hands, but sometimes also the heads, eyes, and other body parts. Gestures are spontaneous and largely unmonitored. Crucially, they are not mere echoes of speech. Temporally, gestures reliably precede the associated speech; semantically, they often complement, rather than duplicate the content of accompanying speech (McNeil, 1992). Gestures, therefore, offer a glimpse into online cognitive processing, a window into thought (Goldin-Meadow, 2003; Goldin-Meadow, Alibali, & Church, 1993). Moreover, gestures are often metaphorical (Cienki & Müller, 2008). We recently observed a friend saying, ‘My mood has really been improving’ while simultaneously tracing an upwards trajectory with her finger. Note that, in this case, the speech contains no hint of a spatial metaphor, but the accompanying gesture reflects the HAPPY IS UP metaphor in which mood is conceptualized as vertical space (e.g. ‘I’m down in the dumps,’ ‘Things are looking up;’ Lakoff & Johnson, 1980). Analysing spontaneous co-speech gesture, therefore, can reveal the presence of embodied, metaphorical thought in the absence of metaphorical speech (Núñez, Cooperrider, Doan, & Wassmann, 2012b).

The spontaneous gestures of non-expert undergraduate students suggest that they conceptualize arithmetic using OBJECT COLLECTION or MOTION ALONG A PATH metaphors (Marghetis, in preparation). Undergraduate volunteers were asked, ‘Can you explain why the sum of an odd number and an even number is always odd?’ and their responses were video-recorded as they reasoned aloud. Just previously, they had read a proof of a related theorem, and completed one of two mental imagery tasks. For participants in the ‘Path’ condition, the mental imagery task involved memorizing a picture of a bead on a wire and then imagining sliding the bead back and forth along the wire. For those in the ‘Collecting’ condition, the picture was of collections of beads, and participants had to imagine combining the different collections. If reasoning about arithmetic relies on an embodied understanding of arithmetic, then participants’ gestures should reflect their use of these metaphors. Moreover, if metaphorical reasoning involves activating the associated source domain, then the mental imagery task should systematically bias participants to reason in a particular

way: using a MOTION ALONG A PATH metaphor after imagining sliding beads along a path, or an OBJECT COLLECTION metaphor after imagining manipulating collections of beads.

This was exactly what was found. Overall, participants deployed two recurring types of gesture – *Collecting* and *Path* gestures – that evoked the two complementary metaphorical construals of arithmetic. In its canonical form, the Collecting gesture consisted of both hands moving inward, with the hands shaped as if they were grasping, pinching, or holding. Morphologically, this type of gesture used volume to evoke numerical magnitude; kinematically, inward motion to evoke addition. Collecting gestures, therefore, suggested an ARITHMETIC IS OBJECT COLLECTION conceptualization (Figure 21.1).

*Path* gestures, meanwhile, used one hand in a canonical pointing handshape to trace motion along a horizontal axis placed slightly in front of the body. Locations along the horizontal axis indicated object identity and magnitude, and motion along the axis indicated arithmetic operations, suggesting a conceptualization of arithmetic using fictive motion and the ARITHMETIC IS MOTION ALONG A PATH metaphor (Figure 21.2).

Participants' gesture suggested that they were conceptualizing arithmetic metaphorically, grounding their understanding in primitive embodied experiences: *Object Collection* and MOTION ALONG A PATH. Moreover, mental imagery had a significant effect on conceptualization, as indexed by gesture. After imagining combining sets of beads, participants were significantly more likely to produce grasping, bimanual *Collection* gestures while

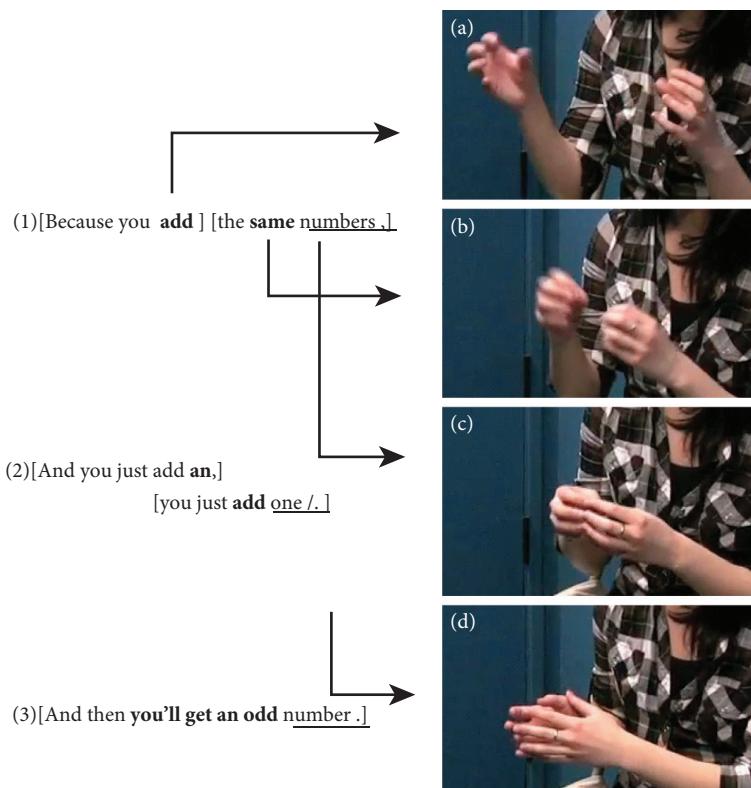
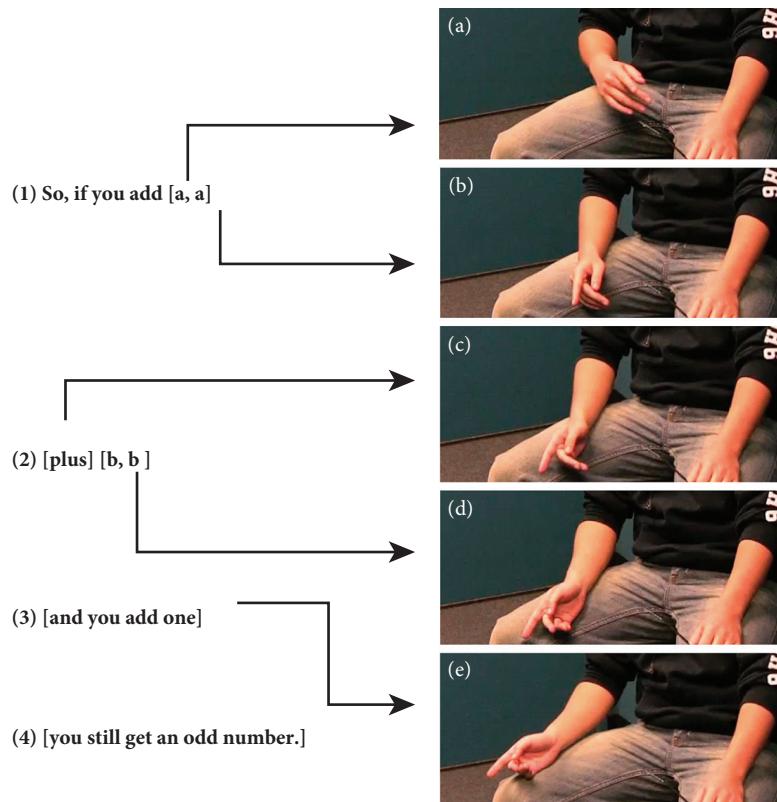


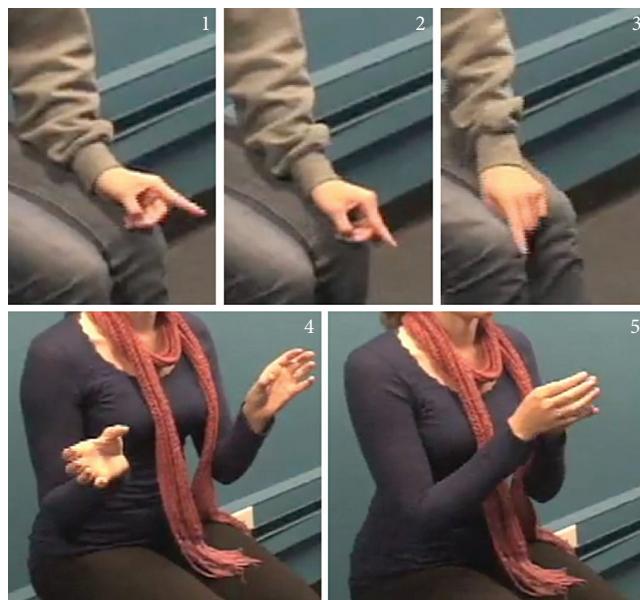
FIGURE 21.1 A canonical *Collecting* gesture.



**FIGURE 21.2** A canonical *Path* gesture. The participant traces a path from left to right. The addition of each term is accompanied by a rightward stroke.

talking about arithmetic. However, if participants imagined moving a bead along a wire, they were more likely to produce *Path* gestures. Recent imagined experience, therefore, systematically primed one metaphor or another, suggesting that reasoning about arithmetic remains grounded in these concrete domains. Moreover, these metaphorical gestures were produced even after an unrelated mental imagery task (memorizing a picture of animals), which shows that participants activate these metaphors spontaneously, even in the absence of thoughts about motion along a path or collections of objects (Figure 21.3). In summary, gesture revealed a spontaneous reliance on metaphorical reasoning, and the choice of a particular metaphor was shaped by recent imagined experience.

Metaphorical gestures about arithmetic are not restricted to naïve experimental participants in the lab, but are produced even by mathematical experts in more naturalistic settings (Marghetis and Núñez, 2013). Mathematics graduate students worked in groups of two to prove a non-trivial theorem in the branch of mathematics known as Analysis. Their interactions were video-recorded, and we analysed the spontaneous co-speech gestures produced by these graduate students as they generated the proof. Using a coding scheme, participants' gestures were coded as 'dynamic' or 'static,' where dynamic gestures profiled movement and paths of motion and static gestures profiled locations, motionless figures, etc. Independently, the content of speech was searched for talk of concepts that, on a cognitive linguistic analysis, should be conceptualized



**FIGURE 21.3** Participants spontaneously produced Path (frames 1–3) and Collecting gestures (frames 4–5) while discussing arithmetic, even without first performing related mental imagery. In the top frames, the participant is saying, ‘Because seven is three plus three plus one.’ In the bottom frames, the participant is saying, ‘And you add your two new even numbers and they make another even number.’

either statically or dynamically – that is, concepts where fictive motion is thought to play a central role. Examples of dynamic concepts included the concepts of INCREASING, FUNCTION INTERSECTION and LIMIT. Crucially, however, none of these concepts are *technically* dynamic; according to their mathematical definitions, they are entirely static, defined in terms of static quantifiers, inequalities, and set notation. For instance, technically, a series of numbers is *increasing* if subsequent numbers are *greater than* preceding numbers – a static notion, which only becomes dynamic if one conceptualizes the numerical change as a [fictive] movement. As predicted, the graduate students were more likely to gesture dynamically when talking about dynamic concepts. For instance, one student was reasoning aloud about the behaviour of a particular function, out of sight of his collaborator, when he noted that the function was increasing. As he said, ‘increasing,’ his right hand shot up from his waist, index finger extended, and traced a leftward and upward trajectory. At the time, the blackboard was covered only in formal symbols, mostly set notation, and his speech was notably absent of metaphorical expressions. This graduate student spontaneously used gesture to spatialize a numerical increase as a motion along a path (for details, see Marghetis & Núñez, 2013). Even the gestures of mathematics graduates students, therefore, reflect the spontaneous use of metaphorical reasoning, construing the real numbers – even more sophisticated than PCNs or the natural numbers – as locations in space, and numerical change as motion through space.

## Discussion

Broad claims about the ‘embodiment’ of mathematics can gloss over the sundry, but specific ways in which numbers and arithmetic might be embodied. In this section, we canvassed the converging evidence that early acquisition of number concepts – PCNs, but also more elaborated concepts like arithmetic – rely on domain-general mechanisms that support human imagination: conceptual metaphor and fictive motion. Moreover, we argued that these mechanisms remain active in mature reasoning about arithmetic, even among experts like graduate students. Conceptual metaphor and fictive motion, it seems, allow us to conceptualize the most abstract concepts in terms of basic, shared human experiences.

## CONCLUSION AND FUTURE PROSPECTS

The field of number cognition investigates the concept(s) of ‘number’ to gain insight into mathematical cognition, and perhaps the nature of mathematics itself. The term number, however, is highly polysemous, covering cases that go from PCNs, which often can be written with single digits, to natural, negative, rational, real, complex, infinitesimal, or transfinite ones. The vast majority of research on number cognition has focused on PCNs and the cognitive pre-conditions for numerical abilities, such as subitizing and approximate discrimination of large numerosity. These pre-conditions, however, are not sufficient to account for the rich inferential organization of PCN concepts, let alone natural numbers and arithmetic. It is crucial that we understand that these pre-conditions are just that, ‘pre-conditions,’ not ‘proto’ or ‘early’ *numerical* abilities – not any more than the vestibular system supporting an infant’s first steps is proto – or early snowboarding (Núñez, 2011).

Humans build on these pre-conditions to move onto counting and PCNs, elaborate those first concepts to get to arithmetic, and eventually construct the astonishing edifice of mathematics. To do so, we have argued, humans use every day cognitive mechanisms for imagination, such as conceptual metaphor and fictive motion. While these mechanisms were first studied by cognitive linguists concerned primarily with language, they appear to be domain-general and ubiquitous in thought. There is accumulating evidence that these metaphors and mapping mechanisms have, beyond the linguistic expressions, a clear psychological and neurological reality – evidence from a variety of domains, ranging from psychological experiments, to gesture studies, to neuroimaging. Of course, much more needs to be understood about the psychological and neural underpinnings of these powerful conceptual mechanisms, but it is now increasingly clear that when it comes to number cognition, they play a role in early acquisition, support more advanced learning, and remain active during mature mathematical cognition.

Our analysis has not even touched the richness and grandeur of the number concepts found in contemporary mathematics: real, complex, infinitesimal, transfinite. We do ourselves a disservice by equating simple concepts of numerosity with counting numbers, or even with natural numbers, let alone the more complex concepts that populate the mathematical universe. Once all the relevant distinctions have been made clear, the numerical cognition of the future must not shy away from investigating the role of language,

culture, and history in the consolidation of these rich, magnificent, and diverse human accomplishment – numbers.

## ENDNOTES

1. As other contributions in this Handbook show, some processing research has investigated multi-digit integers, fractions, and negative numbers as well (see Nuerk et al. this volume). Still, the bulk of number cognition research – shaping the development of the field – has been devoted to PCNs.
2. Like ‘number,’ the term embodied is polysemous. In this chapter, we are primarily interested in the use of ‘embodied’ as the grounding of the inferential structure in schematized bodily experience as when one moves along generic paths or gather generic collections of objects. We will put less emphasis on concrete specific bodily-supported actions such as finger-counting or on the use of ‘embodied’ brain areas during number processing.
3. Spatial construals of time have, of course, many other complexities, which go beyond the scope of this chapter (for a review, see Núñez & Cooperrider (2013)).
4. Throughout, we reserve **SMALL CAPITALS** to denote concepts or conceptual metaphors, to distinguish them from particular linguistic manifestations of these mappings (e.g. ‘She has a great future in front of her’).
5. Note that these differ from the counting principles identified by Gelman and Gallistel (1978), which identify the abstract principles that must be acquired to use a count routine. Here we are concerned with the cognitive prerequisites for applying such a count routine.
6. These results are also consistent with the ATOM proposal (Walsh, 2003), which posits a domain-general representation of magnitude, shared between space, time, number, and even luminance (Cohen Kadosh & Henik, 2006). It’s unclear, however, that pitch is naturally understood as in a single, reliable way: While higher pitches are sometimes associated with higher space (Rusconi, Kwan, Giordano, Umiltà, & Butterworth, 2006), they are also associated with smaller objects or animals (e.g. Ohala, 1994), making it difficult with the ATOM framework to predict a particular association between pitch and number.

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